

A Parallel Simulated Annealing Algorithm for the Optimization TDVRP

Shanshan Cui¹, Zhao Jing¹, Bin Shi²

¹School of Information and Computer Engineering, Chuzhou Vocational and Technical College, 239000, China

²China Railway Fourth Bureau, 230000, China

ABSTRACT

In this paper, parallel algorithm based on multi-core cell processors for solving time-dependent vehicle routing problems has been proposed. The aim is to optimize algorithm performance to a greater extent. The SA algorithm has achieved big data parallelism and improved speed. However, the efficiency of this implementation is still limited, as traditional SA algorithms still run at low parallelism on new platforms and computational resources can not be fully utilized. For these problems, This algorithm creates the initial solution using heuristics, and two editions of parallel algorithm are described, which referred to as independent search and collaborative search. Figliozi's benchmark test covered the calculation results of 56 test questions from 100 customers, the results show that (i) compared with traditional sequential SA, p-SA gains better solutions in different time dependent speeds with an accepted computational time. Hence, through data analysis, P-SA can effectively solve the TDVRP problem, and can be extended to deal with other variants of the open vehicle routing problem and other combinatorial optimization problems. In addition, the proposed algorithm can significantly improve the execution efficiency of simulated annealing processes.

KEYWORDS

TDVRP; parallelizable system; simulated annealing; Solomon's benchmark; multiple Markov chain

1 Introduction

Ramser and Dantzig firstly put forward the vehicle routing problem (VRP) in 1959. Then, many scholars studied the vehicle routing problem with time Windows (VRPTW) ^[1-2], which can be regarded as the abstract of distribution vehicle scheduling. However, VRPTW is mostly based on the static network. But in the actual road network, due to the impact of peak hours, weather changes and sudden accidents and other factors, the parameters (the starting time of the vehicle, the driving speed of the road section and so on) of VRP are always in constant change. Therefore, Beasley proposed the Time Dependent Vehicle Routing Problem (TDVRP) in 1981^[3]. The traditional VRP models are difficult to meet the requirements of high-speed delivery; Moreover, VRPTW problems are the most representative issue in this field due to its wide range of application and internal complexity.

Therefore, Studying the TDVRP problems is of great significance^[4]. Some studies have shown that TDVRP includes NP-hard optimization problems, like Bin packaging and TSP^[5-6]. it is very significant to construct a high effective parallel algorithm. Research on TDVRP problems based on a high effective parallel algorithm is of great importance in logistics distribution system.

Arbelaiz optimizes simulated annealing based on parallel framework. Onbas conducts comprehensive research and application based on SA parallelization^[7-8]. Moghaddam R studied Vehicle routing problems of multi model independent path length. Malandrak and Daskin gave a mixed integer programming model for TDVRP. This paper is taken the strategies of parallelizable communication into account. Meanwhile, the solution of per processor is fixed.

This paper uses the synchronous communication strategies to make parallel. Compare the final status after each determined cycle or each search ending. Each thread is corresponded to a simulated annealing ripple, which runs independently. Meanwhile, fix the constant number of solutions per processor. At last, the model and algorithm are verified and analyzed combining with the example. Computational experiments show that the parallel SA algorithm run up faster than a single-core CPU. Independent and co-operating as a strategy for parallel computing, has enormous potential in improving computing speed.

2 Problem descriptio

The description and mathematical model of TDVRP.

TDVRP is the expansion of VRP, so it is described as follows^[9] (figure 1).

Every trip ends and starts at the central depot (only one central depot).

The weight of each route is less than Q with a fleet of vehicles.

Each customer's needs can only be completed by one vehicle at a time. The customers are serviced at the time window. The vehicle could arrive early, but to wait until proper time.

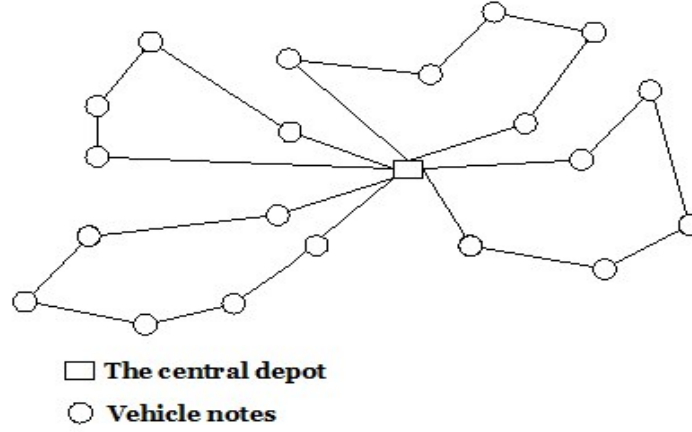


Figure 1 sketch of vehicle routing problem

Under the constrained path, choosing the right path make the total costs minimum, which is the aim. The whole dynamic distribution network is denoted as the net. The static distribution network of time T is $Net(T)$. $Wp(T)$ represents a collection of all key points in T ; $Wpo(T)$ represents a collection of t time all the key points, distribution center and virtual yard; $Wu(T)$ represents a collection of static clients that are not completed and dynamic customer requirements for T moments; $Wuo(T)$ represents that the T moment of all the static customers, the demand for dynamic customer, set user, distribution center and virtual yard; A vehicle fleet is shown as $K = \{1 \dots m\}$; the total number of different models is the letter N . The letter Q_r will be used to indicate the maximum load; $Q_{0kr}(T)$ represents the carrying capacity at T moment (if the vehicle starts from the distribution center, then $Q_{0kr}(T) = 0$); Q_i represents the amount of demand for the i client; L represents the number of customers; 0 expresses the delivery center. C_{ij} represents the distance between a client and a j client (the distance between two points is used in the text); e_{r1} represents the r kind of no-load models and e_{r2} represents the unit fuel consumption cost of full load; P_{ijkr} represents the real load rate model for the R of vehicle K from i customers to the j ; F_r represents the fixed usage cost of the R model; S is a branch elimination constraint set. Decision variable is given:

$$MinZ = \sum_{i \in W_{upo}(t)} \sum_{j \in W_{uo}(t)} \sum_{k=1}^m \sum_{r=1}^n c_{ij} x_{ijk} e_{r1} + \sum_{i \in W_{upo}(t)} \sum_{j \in W_{uo}(t)} \sum_{k=1}^m \sum_{r=1}^n c_{ij} x_{ijk} p_{ijkr} (e_{r2} - e_{r1}) + \sum_{r=1}^n F_r \sum_{i \in W_{upo}(t)} x_{0ikr} \quad (1)$$

$$ST: p_{ijkr} = \left(\sum_{j \in W_{upo}(t)} q_i y_{jkr} - \sum_{i \in W_{upo}(t)} q_i y_{jkr} \right) / Q_r \quad (2)$$

$$ST: p_{ijkr} = \left(\sum_{j \in W_{upo}(t)} q_i y_{jkr} - \sum_{i \in W_{upo}(t)} q_i y_{jkr} \right) / Q_r \quad (3)$$

$$\sum_{i \in W_{uo}(t)} q_i y_{jkr} \leq Q_r - Q_{ijk}(t) j \in W_{po}(t), k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (4)$$

$$\sum_{i \in W_{uo}(t)} \sum_{j \in W_{uo}(t)} x_{ijk} = 0, k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (5)$$

$$\sum_{i \in W_r(t)} \sum_{j \in W_{uo}(t)} x_{ijk} = |W_p|(t), k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (6)$$

$$\sum_{k=1}^m y_{ikr} = 1, i = W_u(t), r = 1, 2, \dots, n \quad (7)$$

$$\sum_{i \in W_{po}(t)} y_{ikr} = m, k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (8)$$

$$\sum_{i \in W_r(t)} x_{ijk} = y_{jkr} j = W_u(t), k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (9)$$

$$\sum_{j \in W_r(t)} x_{ijk} = y_{ikr} i = W_u(t), k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (10)$$

$$\sum_{i \in S} \sum_{j \in S} x_{ijk} \geq 1, \forall S \in W_u(t), k = 1, 2, \dots, m, r = 1, 2, \dots, n \quad (11)$$

A joint distribution model is established to optimize the objective of fixed cost and fuel consumption.

Formulas (1) is the distribution of the total minimum cost objective function, and the first two are the variable cost of using a vehicle, and the latter one is the vehicle' fixed cost; Formulas (2) indicates the actual load rate of vehicle R from customer i to customer j ; Formulas (3) represents that the amount of the vehicle borne by the vehicle R of the K is not greater than the carrying capacity of the vehicle; Formulas (4) and (5) ensure that the access to critical points is one way, and the key point of the vehicle can only be issued, and then return to the virtual yard; Formulas (6) ensures that each customer is only a car service; Formulas (7) ensures that the mount of the cars from the key issue point and virtual yard is

M; Formula (8) and (9) ensure that the vehicle entering the customer service must be out of the customer, which is the relationship between the two variables; Formula (10-11) indicates that the branch elimination constraint, that is, elimination constitutes an incomplete line route.

3 Sequential simulated annealing

SA^[10-13] is a random search algorithm that mimics the solid-state annealing crystallization process. During the optimization process, the optimal solution is accepted, and non optimal solutions may also be accepted according to the algorithm's needs, making the algorithm less prone to falling into local optima Excellent. To settle the TDVRP problem, a two-portions is proposed in the chain(S), including minimized number of vehicles and minimized distance of total driving. Decreasing annealing temperature is reduced as $T_{k+1} = \beta_f T_k$, for the letter $k=0, 1 \cdots a_p$ and β_f ($\beta_f < 1$) and a_f ($f=1$ and 2 express part 1 and 2).

The basic thought of SA (table 1):

Table 1 the procedure of simulated annealing

(1) The letter t is initial temperature (large enough), The letter s is initial state (the origin point of the iterative), and the letter L is the length of each iteration;
(2) For $k=1, \dots, L$; When $k=1, \dots, L$; perform steps (3) to (6);
(3) To generate S' , the letter S' is new explanation;
(4) work out the incremental ($\Delta t' = C(S') - C(S)$), where $c(s)$ is cost-function;
(5) If $\Delta t'$ is less than 0, accept S' as new current solution; otherwise, assume rate $\exp(-\Delta t'/T)$ accepts S' as the new current solution;
(6) If the termination condition is content, export the current solution as the best Solution, algorithm terminated, otherwise return to step (2);
(7) The letter T gradually diminished, and $(T -)$ is more than 0, and then goes to the step (2).

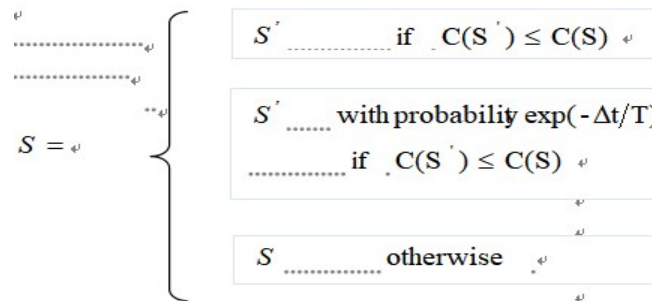


Figure 2 the solution of the update condition

4 Parallel simulated annealing

Using p-SA algorithm, improve the calculation speed of the SA algorithm. It not only ensures that the entire algorithm has large-scale parallel granularity, but also enables the results of previous threads to be passed on to the next thread, realizing the fusion of information search. The process of simulated annealing algorithm based on parallel strategy for solving TDVRP problem displayed in Figure 3.

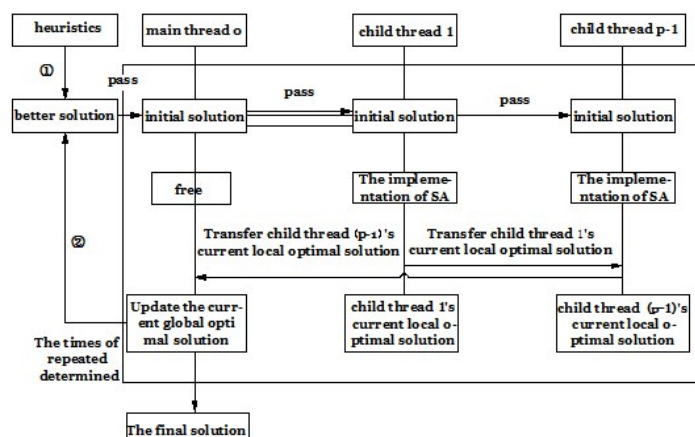


Figure3 parallel simulated annealing algorithm

According to P-SA process, the pseudo-code for solving TDVRP^[14-16] problem can be expressed as shown in algorithm (1-3).

Algorithm 1. parallel simulated annealing algorithm of co-operating searches

```

1 :Parfor Pi, l=0,1,...,p-1 do
2: According to different test sets, use synchronous and Asynchronous communication strategies to create process communication models//
   (Inter thread communication model)
3 :L:=(5E)/p;{The length of the generated cooling chain; E=105}
4: Using heuristic algorithms to create initial solutions// (Set initial solution)
5: current solution s:=initial solution; best solution s':=initial solution;
6: for f= 1 to 2 do {execute phase 1 and 2} {beginning of phase f} // (A process generates its own annealing chain divided into two phases, cool-
   ing stages and annealing steps.) (lines 6-19 )
7: T:=T0,f;{initial temperature of annealing}
8: repeat{a cooling stage}
9:   for i:= 1 to L do
10:    annealing step f (current solution j, best solution j);
11: end for; //(The processes co-operate with each other every  $\omega$  annealing step passing their best solutions found to date (lines 12-16 ))
12: if (f=1) or (co-operation mode is regular) then { $\omega=L$ } // W is a time interval.
13:   cooperation;
14: else{rare co-operation:  $\omega=E$ }
15: Call cooperation procedure every E annealing step counting from the beginning of the phase;
16: end if;
17: T:= $\beta$ f T;{temperature reduction} //(  $\beta < 1$ )
18: until af cooling stages are executed; // af (f=1 and 2 denote phase 1 and 2)
19: end for;
20: end parfor;
21: Produce best_solutionp-1 as the solution to the TDVRP;

```

Algorithm 2. annealing step procedure

```

1: procedure annealing step f(current solution ,best solution );
2: Create new solution as a neighbor to current solution(the way this step is executed depends on f);
3 : $\delta := C_f$  (new solution) -  $C_f$  (current solution);//
4: Generate random x uniformly in the range(0, 1);
5 :if ( $\delta < 0$ ) or ( $x < e^{-\delta/T}$ ) then
6 :current solution:=new solution ;
7: if  $C_f$  (new solution) <  $C_f$  (best solution) then
8: best solution:=new solution;
9: end if;
10: end if;
11: end annealing step f;

```

Algorithm 3. procedure of co-operation of processes

```

1: procedure cooperation;
2 :if j=0 then Send best solution 0 to process P1;
3: else{j>0}
4: receive best solution j-1 from process Pj-1;
5: if  $C_f$  (best solution j-1) <  $C_f$  (best solution j) then
6: best solution j:=best solution j-1;
7: current solution j:=best solution j-1;
8: end if;
9: if j<p-1 then Send best solution j to process Pj+1; end if;
10: end if;
11: end cooperation;

```

Considering Eq. (15), the longer the chain, the better the result .With regard to the p-SA, thread pp-1 need to exchange n steps. For each step of exchanging information, there are (p-1)-1 messages for O (n) being sent and received. It gains time complexity is $O(n^3 + (p-1)*n^2)$.

5 Experimental study

Evaluate the optimization performance of parallel algorithms under different parallel strategies. Using the Solomon

dataset as model experimental data to measure the efficiency of parallel algorithms. This benchmark consists of three datasets, including R, C, and RC topological distributions.

If a processor is used, the totality of calculated programme is S , and having p processors, the total number of calculated solutions is $p \cdot S$. However, the total time required to execute the entire algorithm will remain unchanged. The solutions of S may be taken as 2048 (2K), 4096 (4K), 8192 (8K), 16384 (16K). In this paper, in this case the value of S chosen are 4K (4096) and 8192 (8K), and the number of processors proved, p , are 1, 2, 4, 8 and 16, after 30 model experiments, get general trends in three test sets. The cost function ($f(s)$) with the general trend of increasing processors is shown in figure 4.

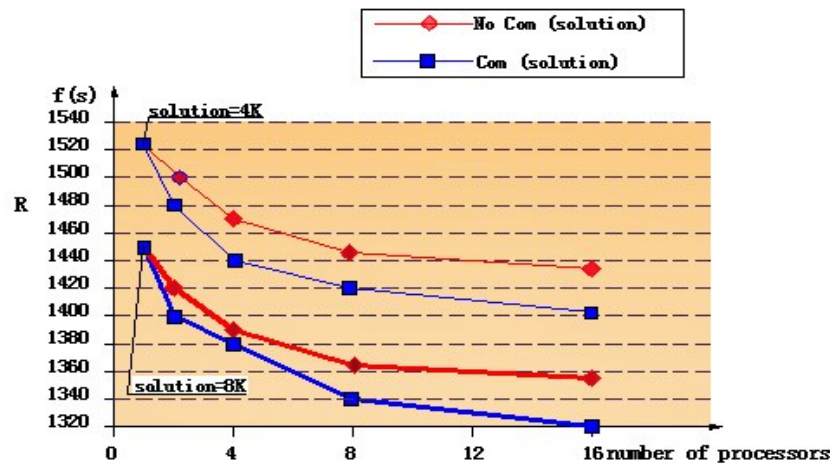


Figure4 Result obtained by changing the quantity of processors on R set

As expected, the impact of adding the quantity of processors is positive in each test set. Therefore, it is concluded that performance is optimized as the quantity of processors raises. Meanwhile, the impact of communication is more positive in every test set than no communication.

6 Conclusions

The application of this VRP is not only in the field of logistics and distribution, but in aviation, ocean shipping and other areas (such as communications, electric power, and industrial management and computer applications). TDVRP is one of the very wide ranges of NP-hard combinatorial optimization problems. According to the nature of TDVRP, the classic serial simulated annealing algorithm is improved. P-SA has been proved to be effective theoretically and experimentally. Along with the development of parallel theory and the complexity of the scale of the TDVRP, there will be much more issue come out and waiting for us to study and explore. Therefore, carry out extensive and in-depth research on vehicle routing problem, which can contribute to the development of operational research studies. The results of their research should be used in the field of logistics delivery and traffic management, which improves operational efficiency and saves on shipping costs. It is of important the oretical value and practical meaning.

Funding

A MapReduce parallel simulated annealing algorithm and its application in VRPTW (YJY-2021-09).

References

- [1] Li Jing Tan, Fu Yong Lin, Hong Wang. BFO Optimization Algorithms for Vehicle Routing Problem with Time Windows[J]. Applied Mechanics & Materials, 2014, 543-547:1884-1887.
- [2] Battarra M, Monaci M, Vigo D. An adaptive guidance approach for the heuristic solution of a minimum multiple trip vehicle routing problem. Comput Oper Res 2009;36(11):3041-50.
- [3] Prins C. A simple and effective evolutionary algorithm for the vehicle routing problem. Comput Oper Res 2004;31(12):1985-2002.
- [4] Vidal T, Crainic TG, Gendreau M, Prins C. A hybrid genetic algorithm with adaptive diversity management for a large class of vehicle routing problems with time windows. Comput Oper Res 2013;40(1):475-89.
- [5] Yusuf, Ismail. Applied genetic algorithm for solving rich VRP [J]. Applied Artificial Intelligence, 2014: 957-991.
- [6] Nagata Y, Bräysy O, Dullaert W. A penalty-based edge assembly memetic algorithm for the vehicle routing problem with time windows. Comput Oper Res 2010;37(4):724-37.
- [7] Onbas, Oglu, E. Parallel simulated annealing algorithms in global optimization [J]. Journal of Global Optimization, 2001: 27-50.

- [8] Czech, Z.J., Co-operation of processes in parallel simulated annealing, Proc. of the 18th IASTED International Conference on Parallel and Distributed Computing and Systems (PDCS 2007), (November 13-15, 2006), Dallas, Texas, USA.
- [9] Tlili T , Krichen S . A simulated annealing-based recommender system for solving the tourist trip design problem[J]. Expert Systems with Application, 2021(Dec.):186.
- [10] Moghaddam R. T. A hybrid simulated annealing for capacitated vehicle routing problems with the dependent route length [J]. Applied Mathematics and Computation, 2006, 176: 445-454.
- [11] He, Guo-Xian. Application of annealing algorithm based on random simulated solution in distribution VRP for single logistics center [J]. Journal of the China Railway Society, 2009: 92-97.
- [12] Da-Wei Zhu, Zhi Qiang. Simulated annealing algorithm for vehicle routing problem [J]. China Journal of Highway and Transport, 2006: 123-126.
- [13] Majid Soheili, Amir Masoud Eftekhari-Moghadam DQPFS: Distributed quadratic programming based feature selection for big data[J] Journal of Parallel and Distributed Computing, 2020, 138(C).
- [14] Alazzam, Hadeel, AbuAlghanam, Orieb, Sharieh, Ahmad Best path in mountain environment based on parallel A* algorithm and Apache Spark[J] The Journal of Supercomputing, 2021(prepublish).
- [15] Shi S M , Wang X F , Zhong F . The research of typical control strategy of electronic power transformer in parallel operation[C]// 2016 IEEE Advanced Information Management, Communicates, Electronic and Automation Control Conference (IMCEC). IEEE, 2016.
- [16] Blocho M , Nalepa J . A Parallel Algorithm for Minimizing the Fleet Size in the Pickup and Delivery Problem with Time Windows[C]// European Mpi Users Group Meeting. 2015.